Interactive Formal Verification
2: Isabelle Theories

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University of Cambridge

A Tiny Theory

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theory BT imports Main begin

datatype 'a bt =
    Lf
  | Br 'a  "'a bt"  "'a bt"

fun reflect :: "'a bt => 'a bt" where
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Notes on Theory Structure

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- The various declarations and proofs may otherwise appear in any order.
- Many declarations can be confined to *local scopes*.
- A finished theory can be imported by others.

Some Fancy Type Declarations

```
typedecl loc  -- "an unspecified type of locations"

type_synonym val   = nat -- "values"
type_synonym state = "loc => val"
type_synonym aexp  = "state => val"
type_synonym bexp  = "state => bool"  -- "functions on states"

datatype
  com = SKIP
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recursive type of commands

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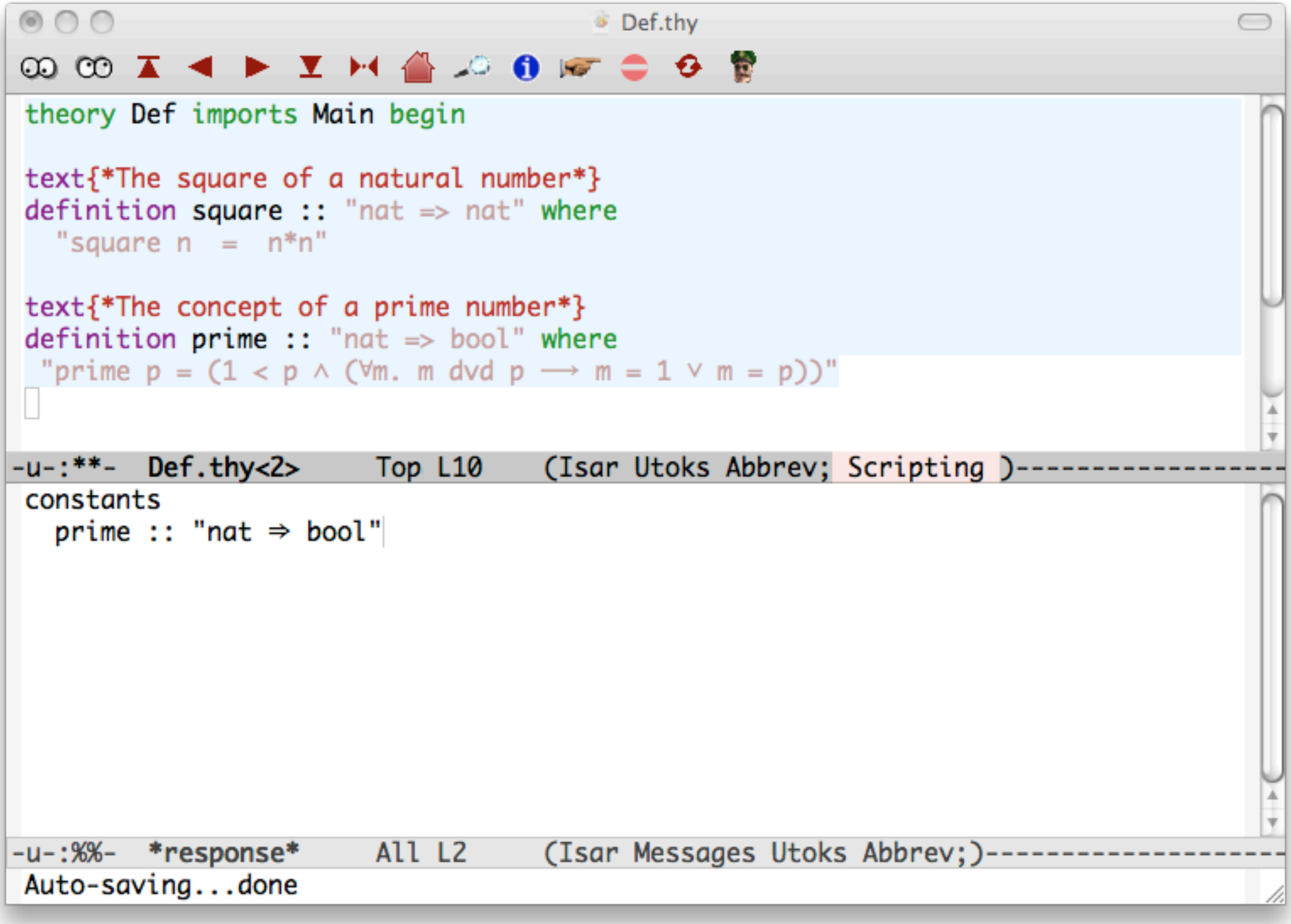
- Type synonyms merely introduce *abbreviations*.
- Recursive data types are less general than in functional programming languages.
  - No recursion into the domain of a function.
  - Mutually recursive definitions can be tricky.
- Recursive types are equipped with proof methods for *induction* and *case analysis*.

Basic Constant Definitions



```
theory Def imports Main begin

text{*The square of a natural number*}
definition square :: "nat => nat" where
  "square n  =  n*n"

text{*The concept of a prime number*}
definition prime :: "nat => bool" where
 "prime p = (1 < p ∧ (∀m. m dvd p ⟶ m = 1 ∨ m = p))"

-u-:**-  Def.thy<2>     Top L10     (Isar Utoks Abbrev; Scripting )-----------------
constants
  prime :: "nat ⇒ bool"

-u-:%%-  *response*     All L2      (Isar Messages Utoks Abbrev;)------------------
Auto-saving...done
```

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- Every variable on the right-hand side must also appear on the left.
- In proofs, definitions are *not* expanded by default!
  - Defining the constant *C* to denote *t* yields the theorem *C*`_def`, asserting *C*=*t*.
  - Abbreviations can be declared through a separate mechanism.

Lists in Isabelle

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- The standard Isabelle environment has a *comprehensive list library*:
  - Functions # (cons), `@` (append), `map`, `filter`, `nth`, `take`, `drop`, `takeWhile`, `dropWhile`, ...
  - Cases: (`case` xs `of` [] ⇒ [] | x#xs ⇒ ...)
  - Over 600 theorems!

List Induction Principle

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To show $\varphi(xs)$, it suffices to show the *base case* and *inductive step*:

- $\varphi(\mathrm{Nil})$
- $\varphi(xs) \Rightarrow \varphi(\mathrm{Cons}(x,xs))$

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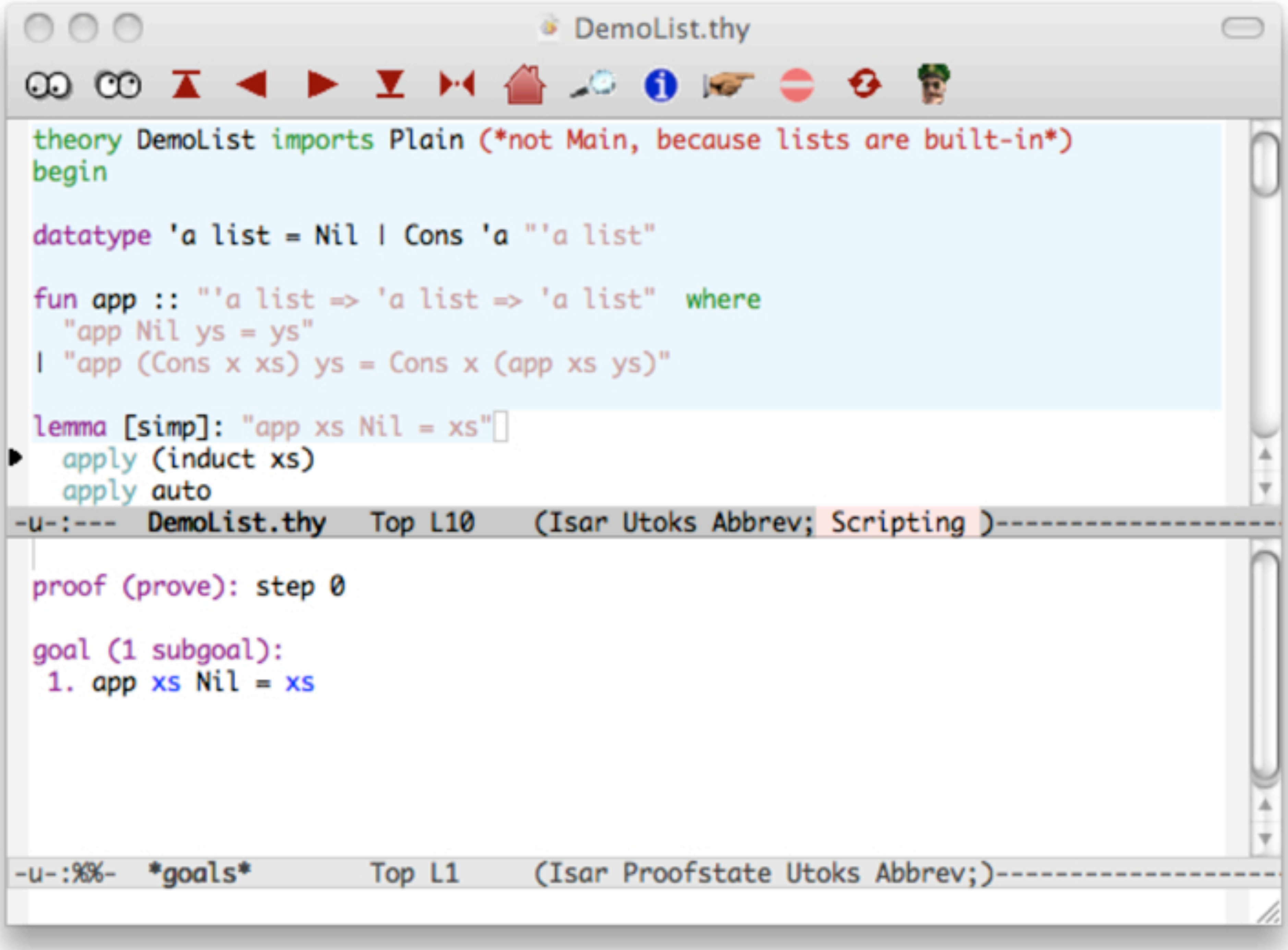
To show φ(*xs*), it suffices to show the *base case* and *inductive step*:

- φ(Nil)
- φ(*xs*) ⇒ φ(Cons(*x*,*xs*))

The principle of case analysis is similar, expressing that any list has one of the forms Nil or Cons(*x*,*xs*) (for some *x* and *xs*).

Proof General



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begin

datatype 'a list = Nil | Cons 'a "'a list"

fun app :: "'a list => 'a list => 'a list"  where
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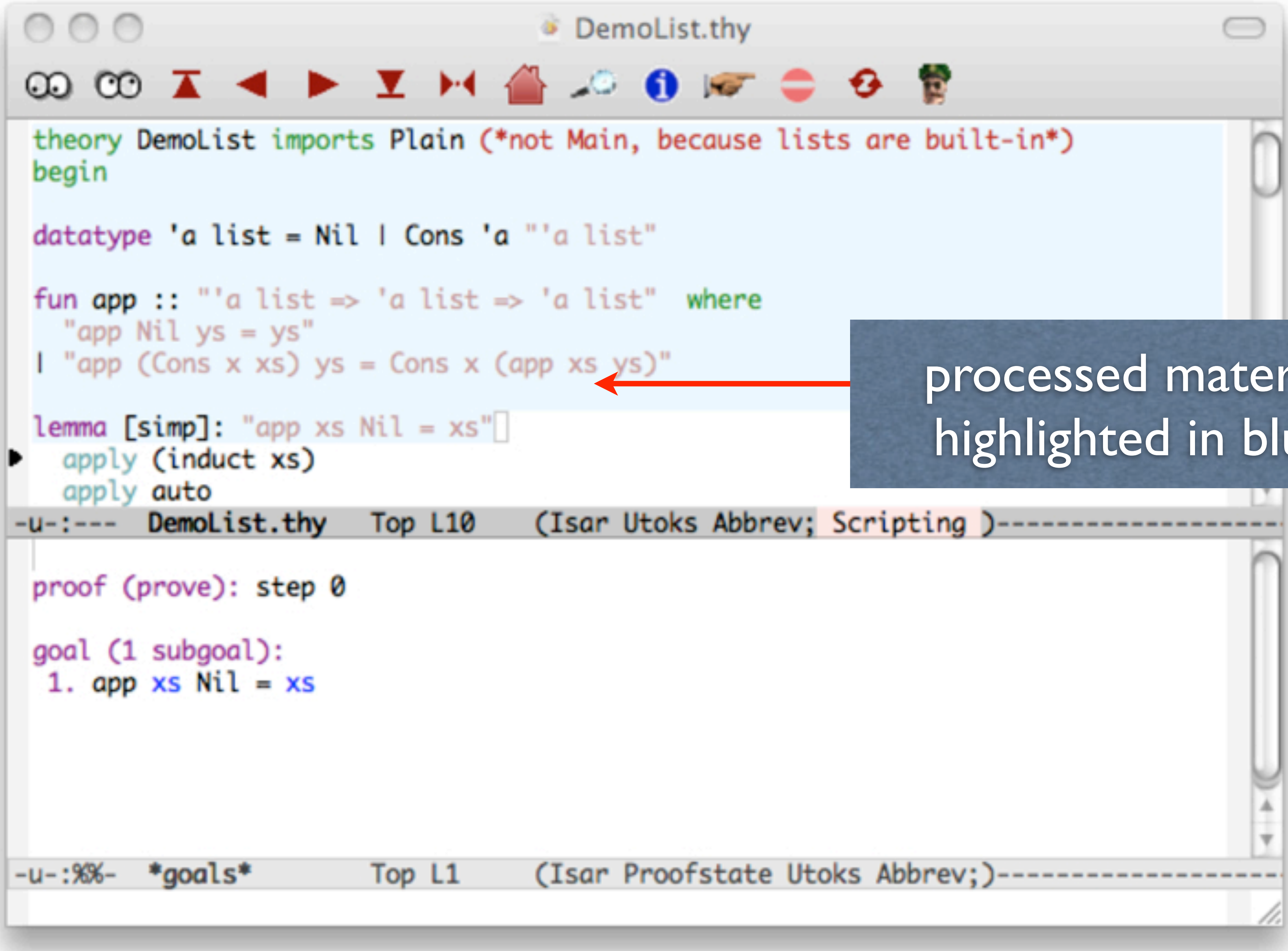

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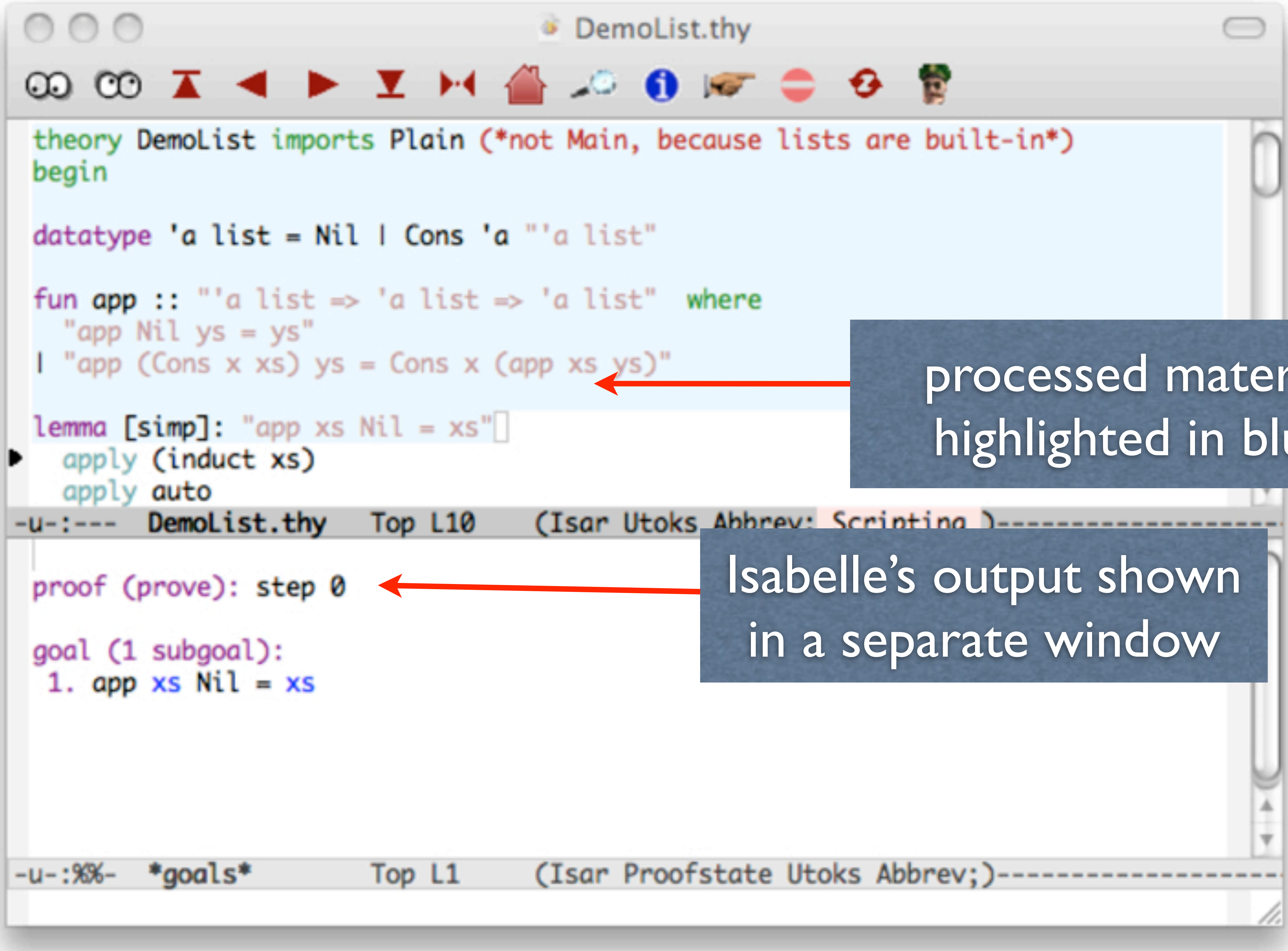

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Isabelle's output shown in a separate window

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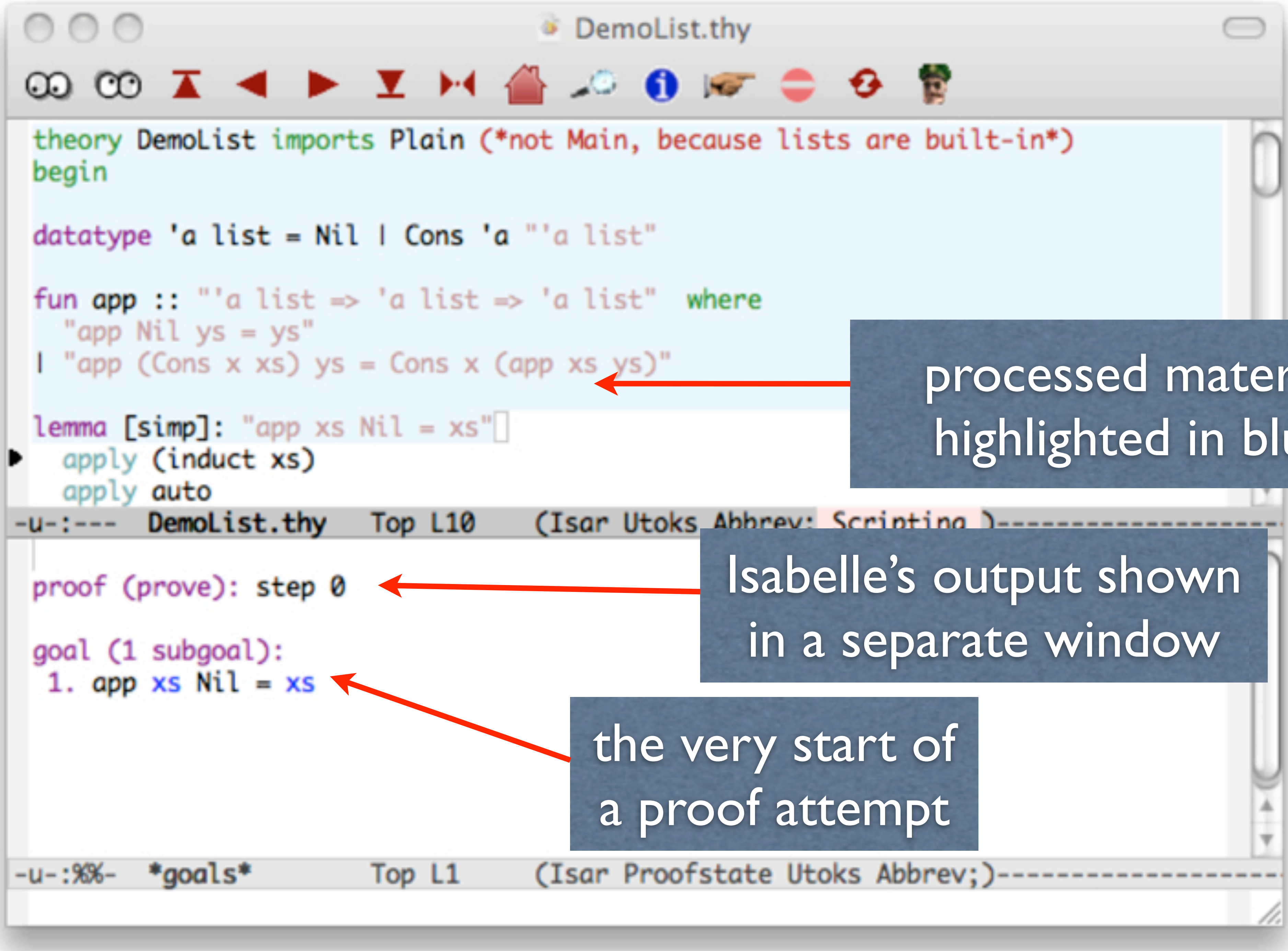

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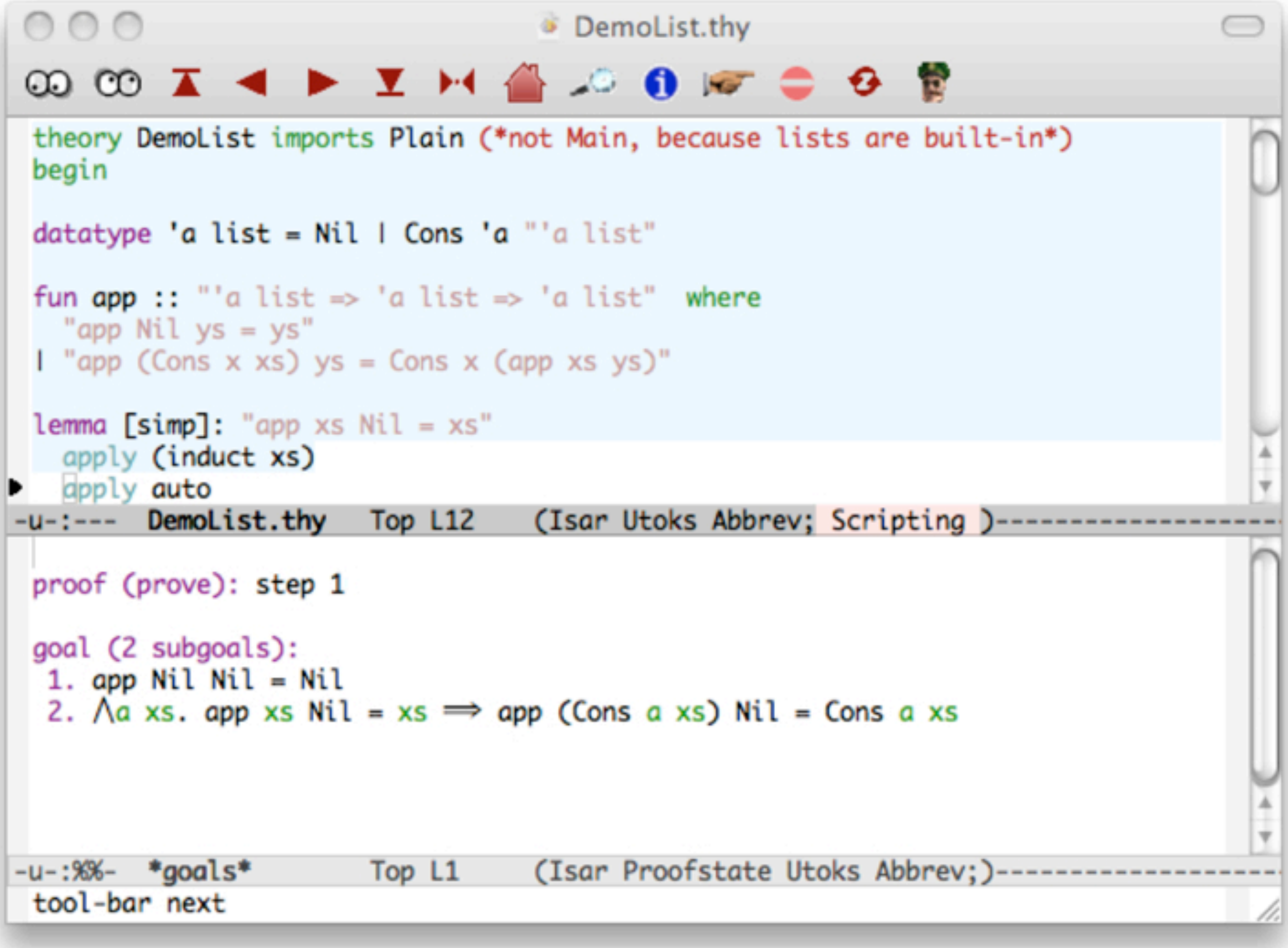
the very start of a proof attempt

Proof by Induction



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goal (2 subgoals):
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tool-bar next
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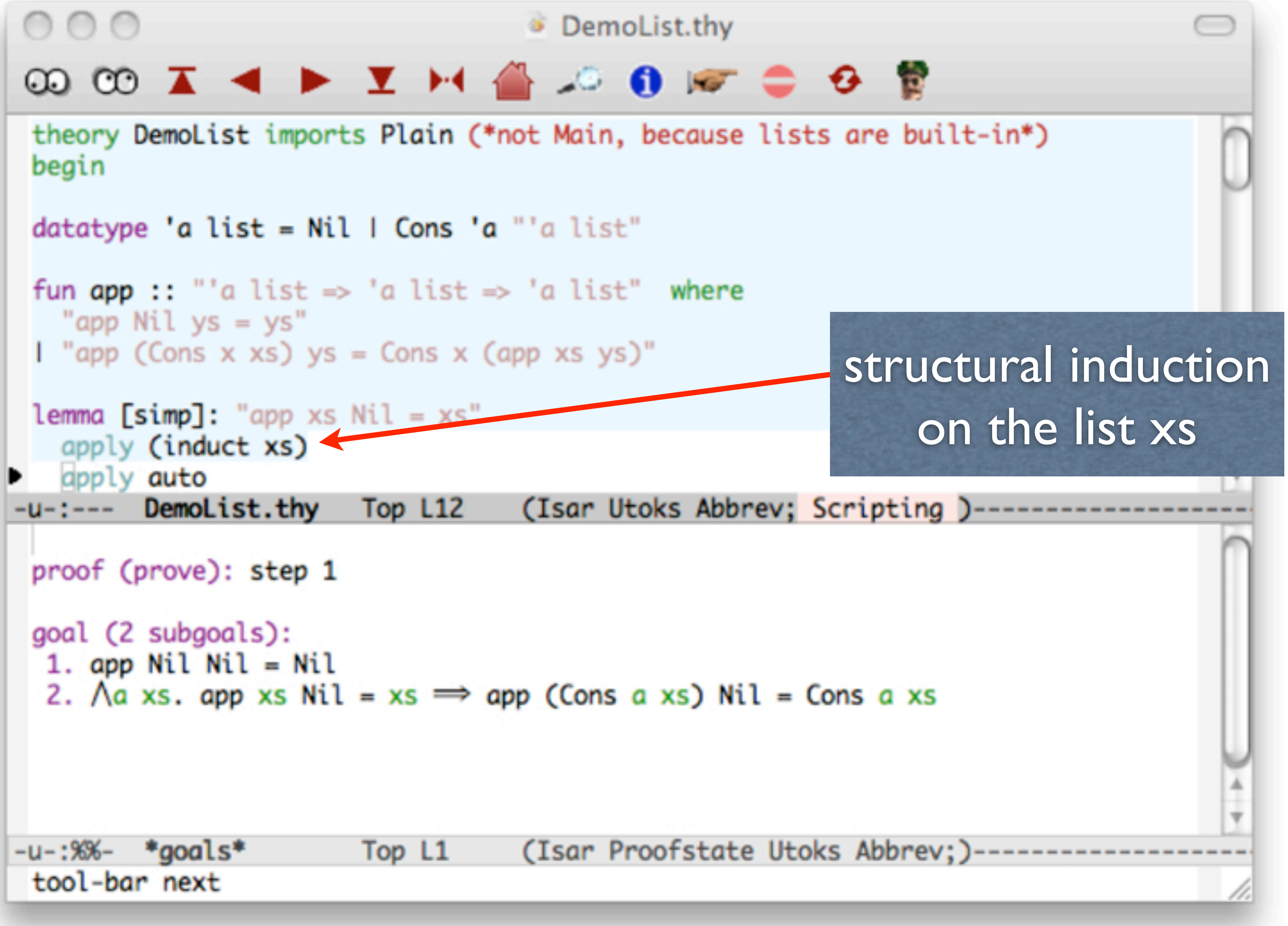

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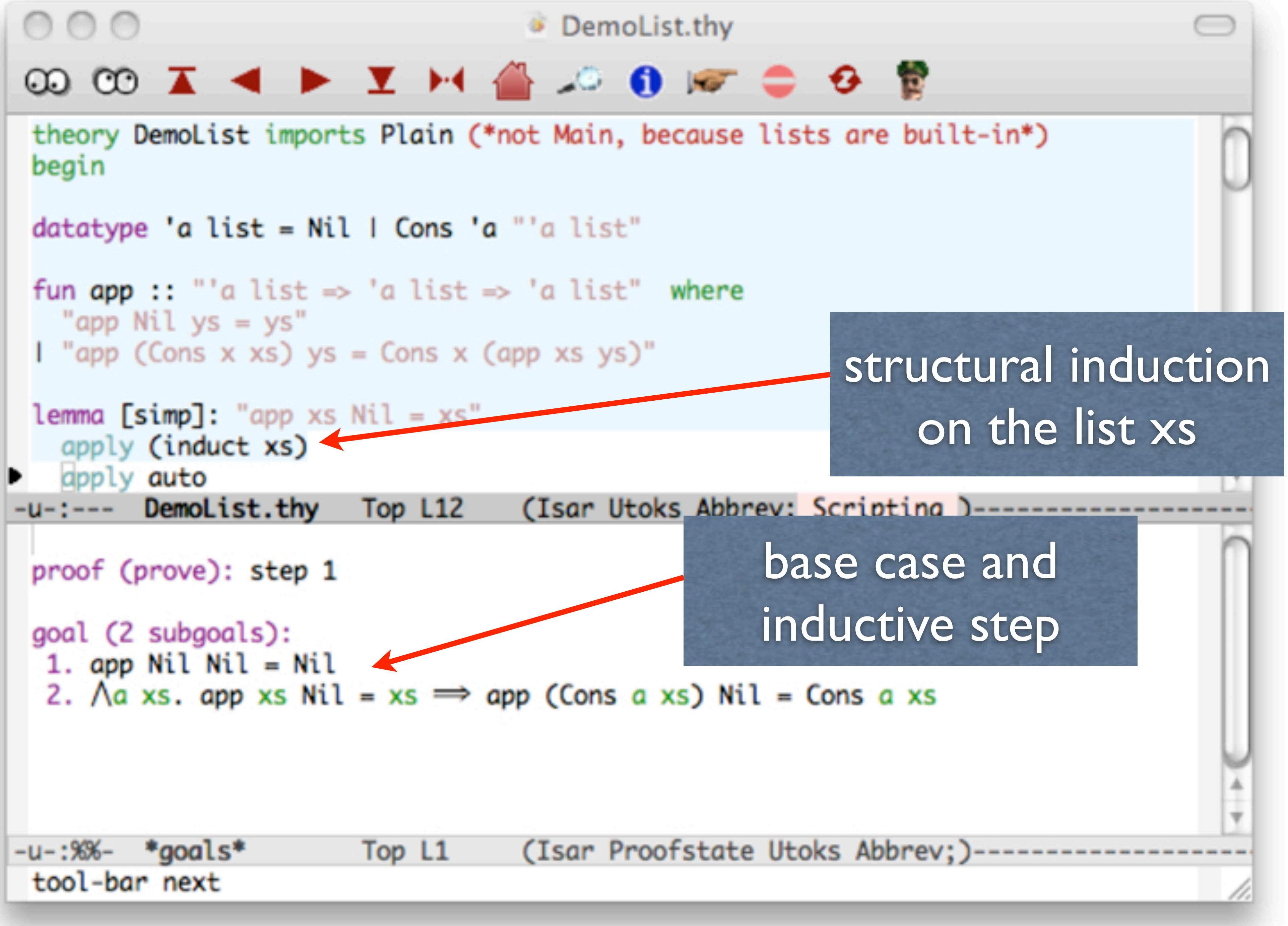

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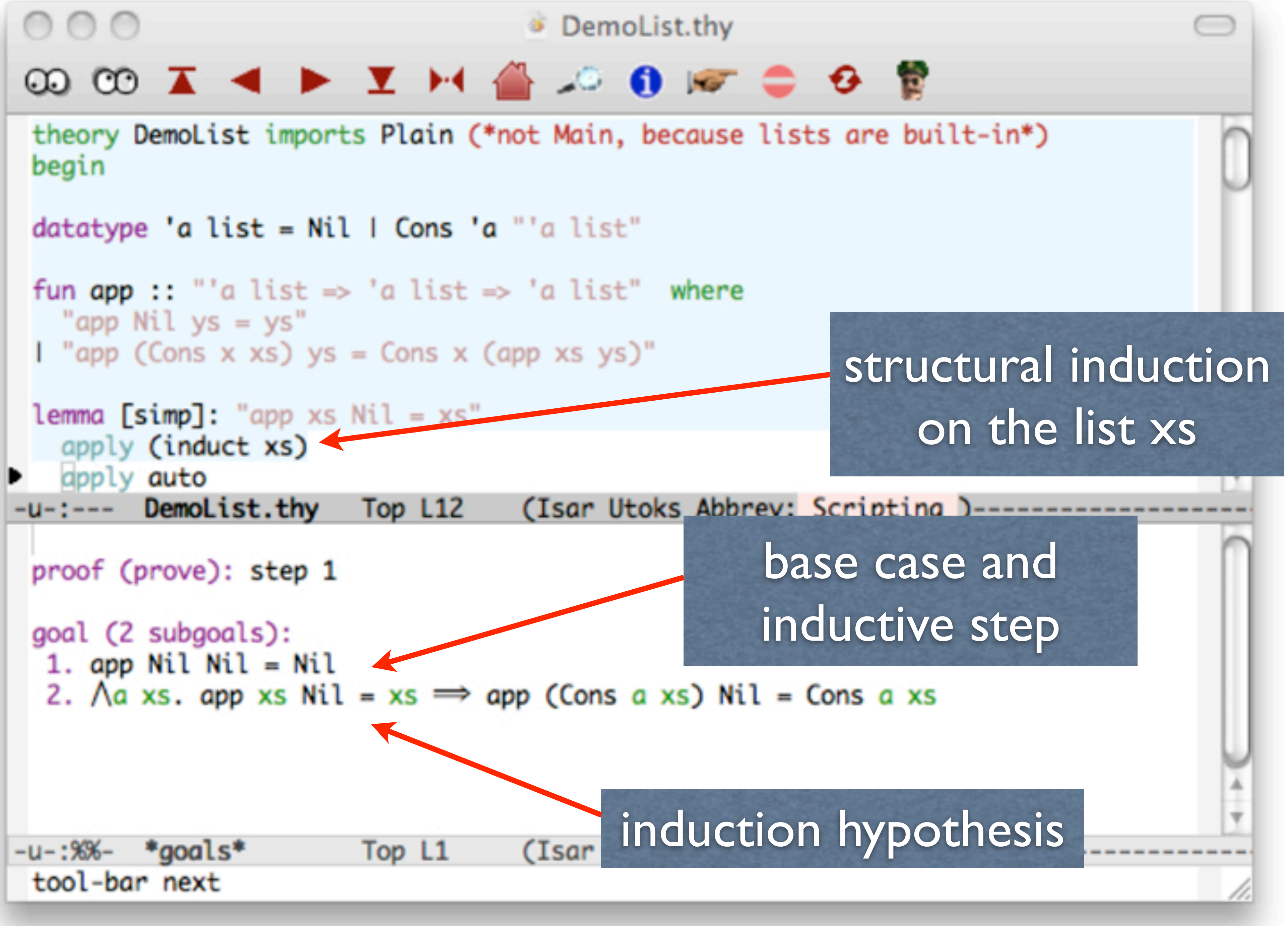

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structural induction on the list xs

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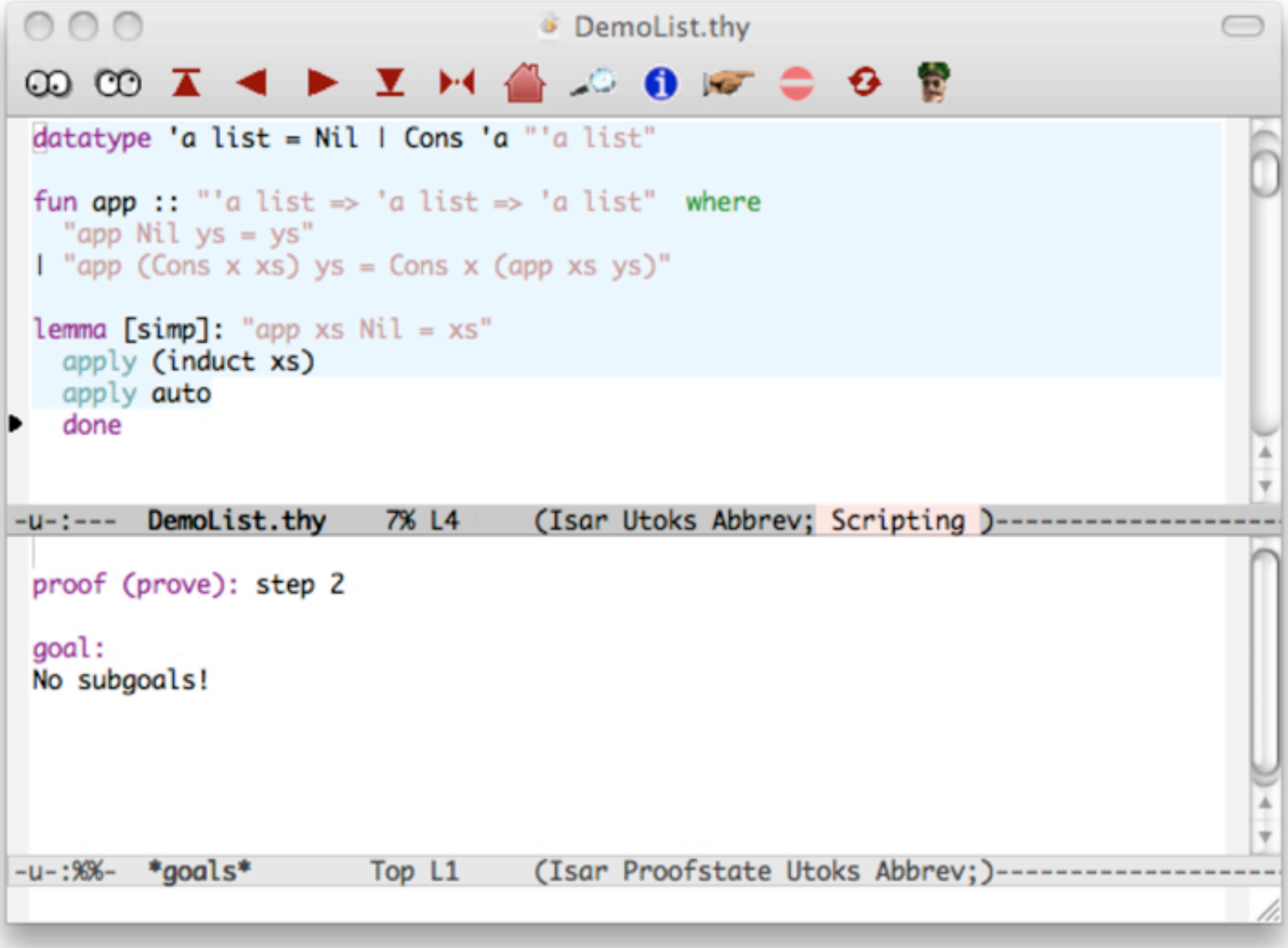
induction hypothesis

Finishing a Proof



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proof (prove): step 2

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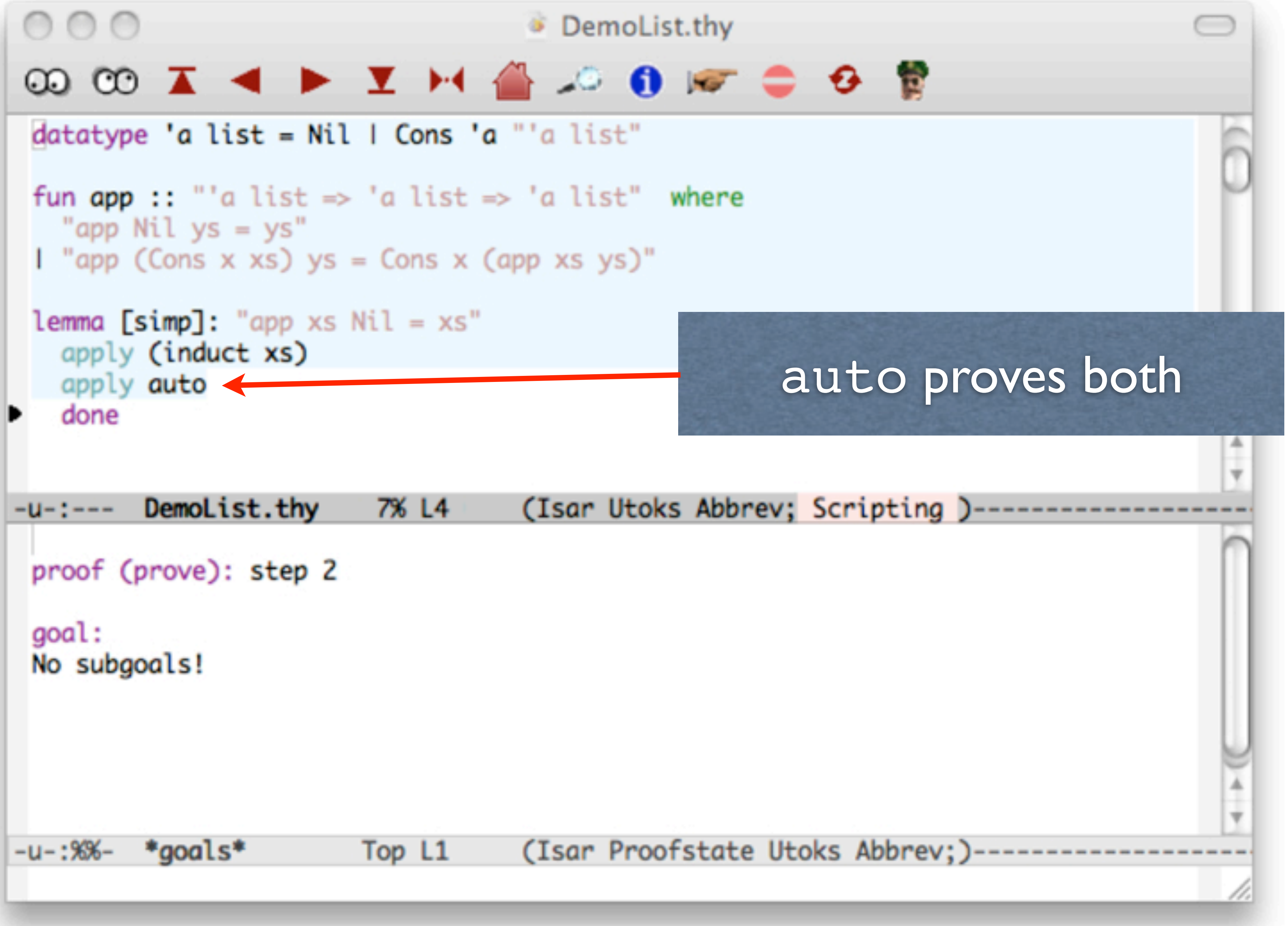

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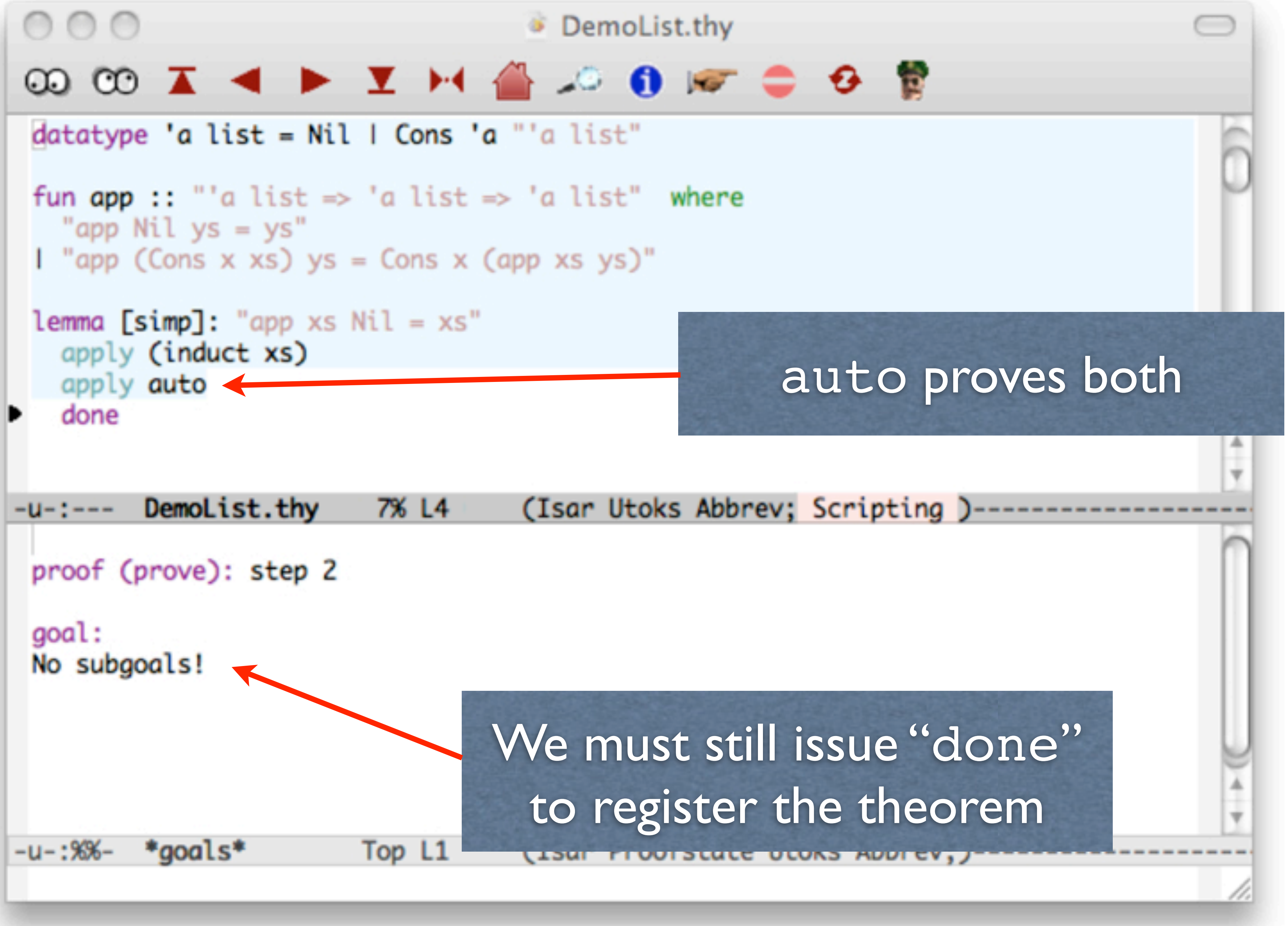

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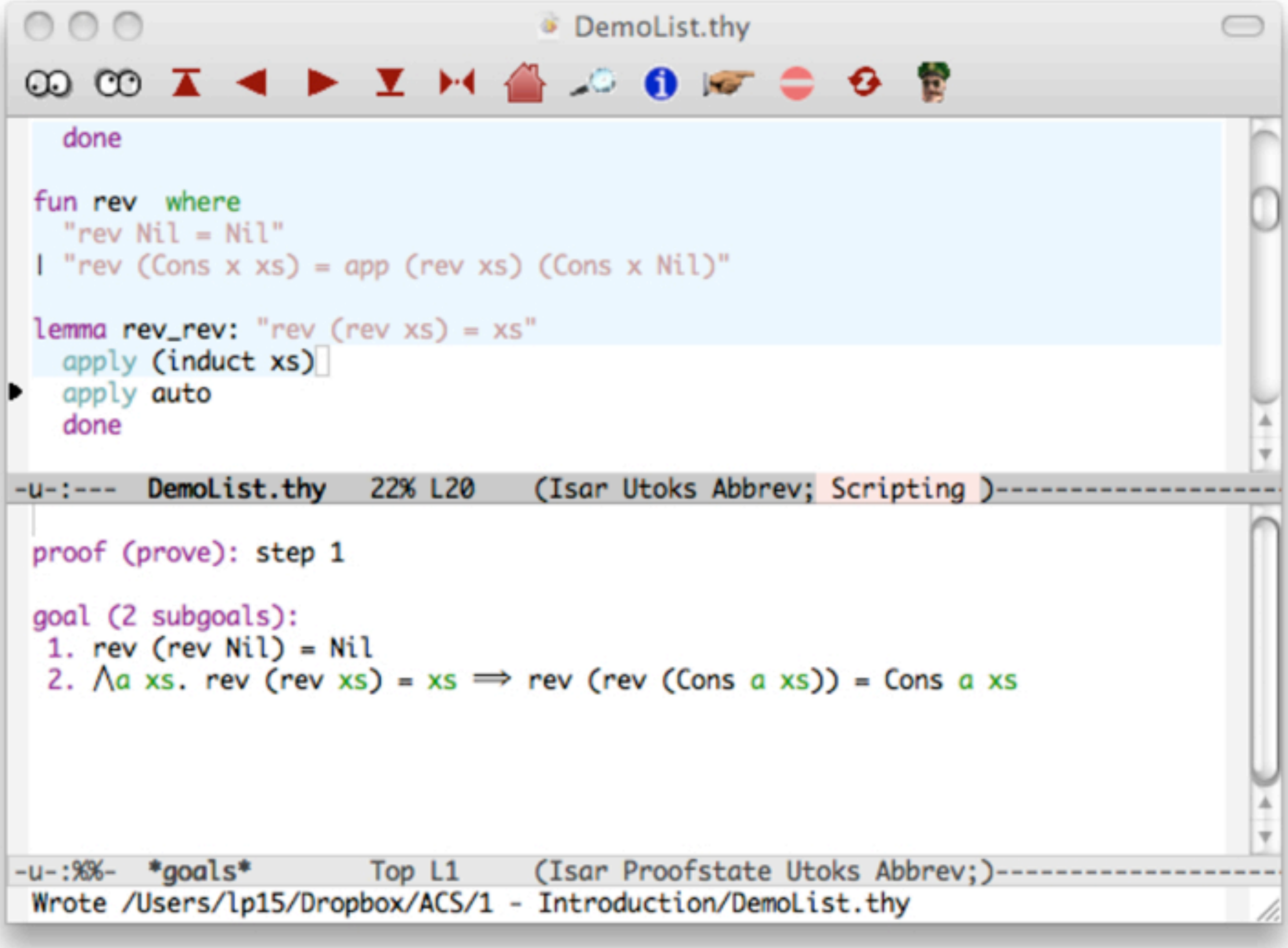
We must still issue "done" to register the theorem

Another Proof Attempt



```
  done

fun rev  where
  "rev Nil = Nil"
| "rev (Cons x xs) = app (rev xs) (Cons x Nil)"

lemma rev_rev: "rev (rev xs) = xs"
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```
-u-:---  DemoList.thy    22% L20    (Isar Utoks Abbrev; Scripting )------------------
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proof (prove): step 1

goal (2 subgoals):
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```

```
-u-:%%-  *goals*         Top L1     (Isar Proofstate Utoks Abbrev;)------------------
Wrote /Users/lp15/Dropbox/ACS/1 - Introduction/DemoList.thy
```

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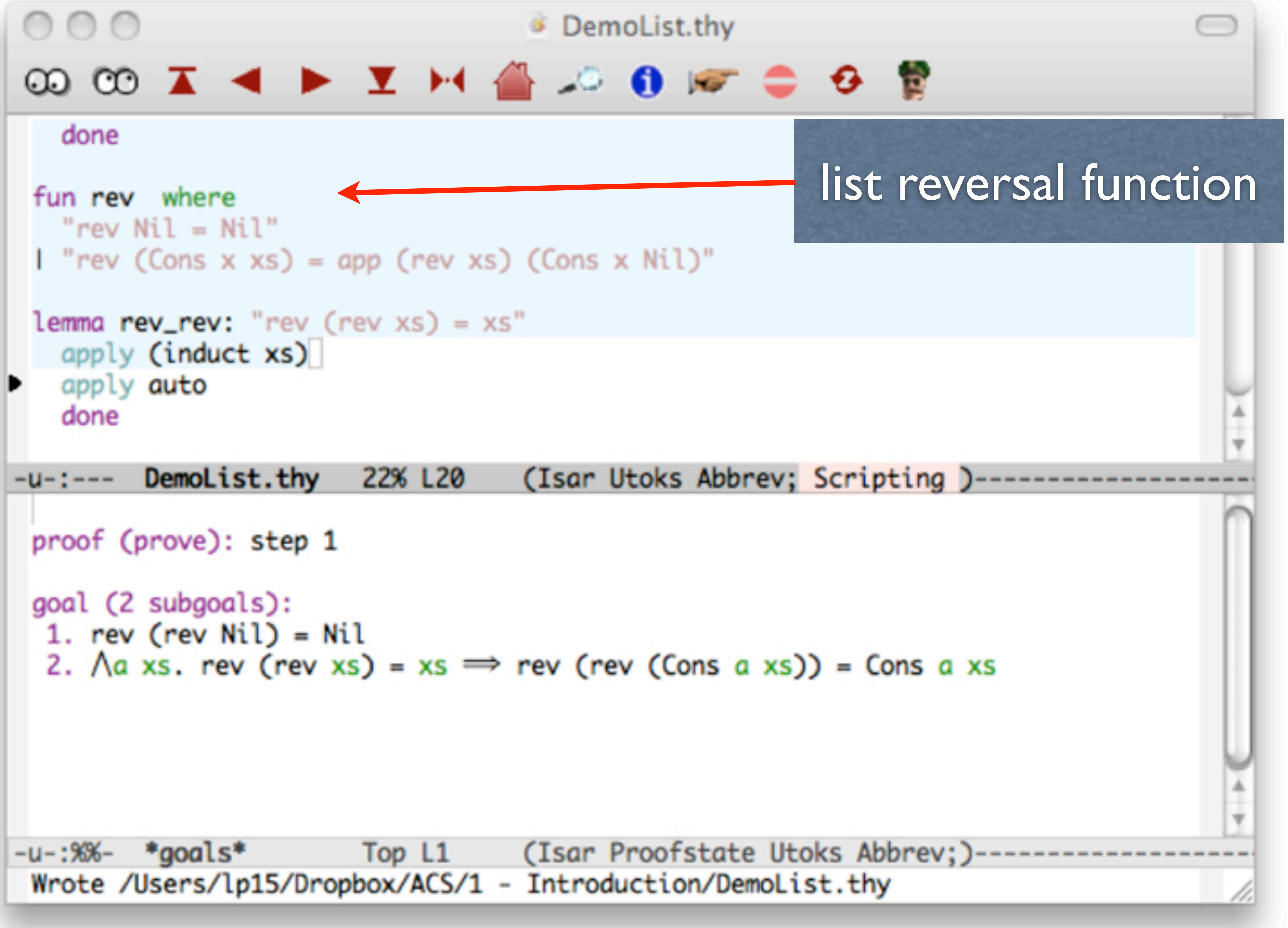

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list reversal function

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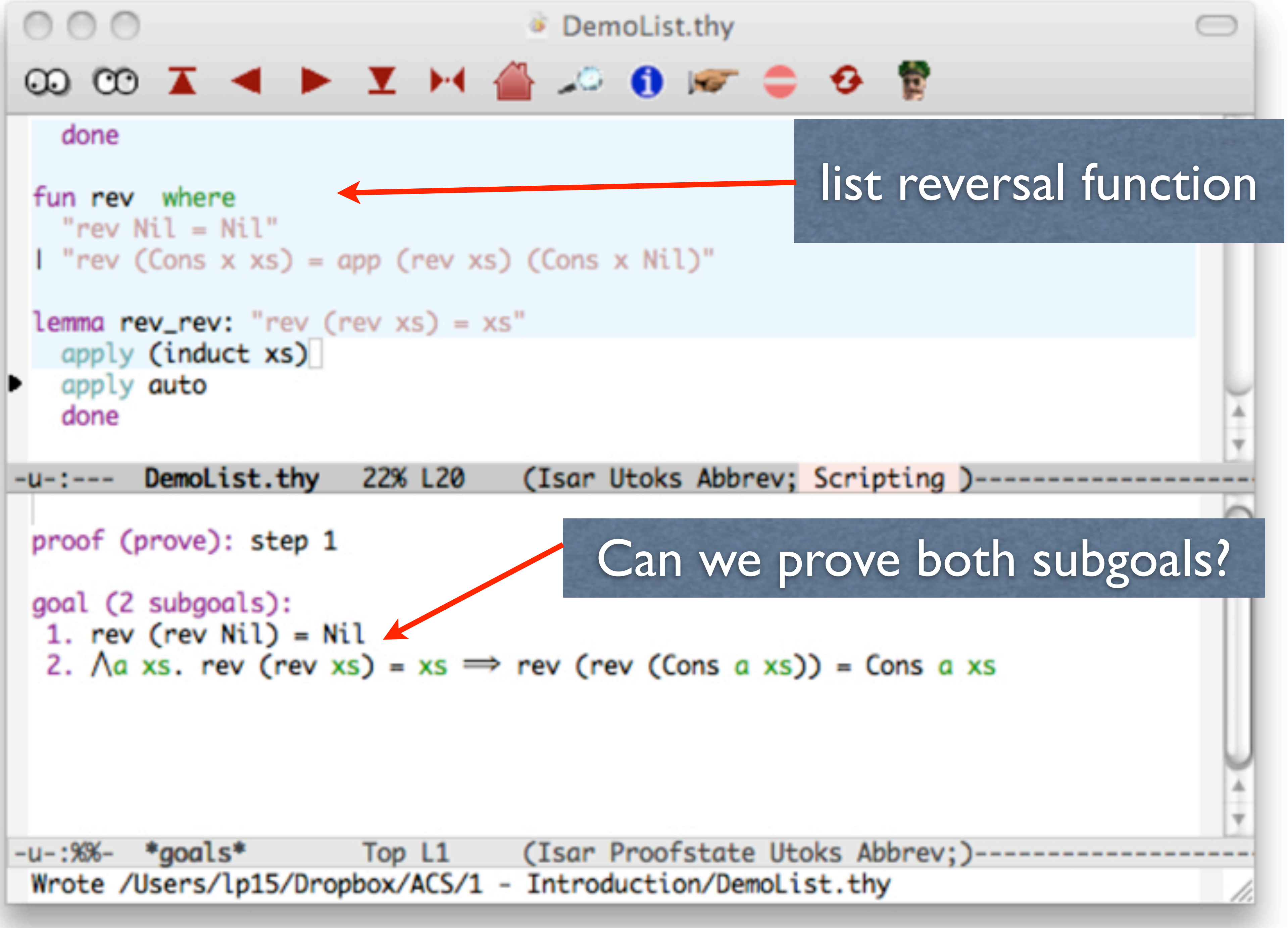

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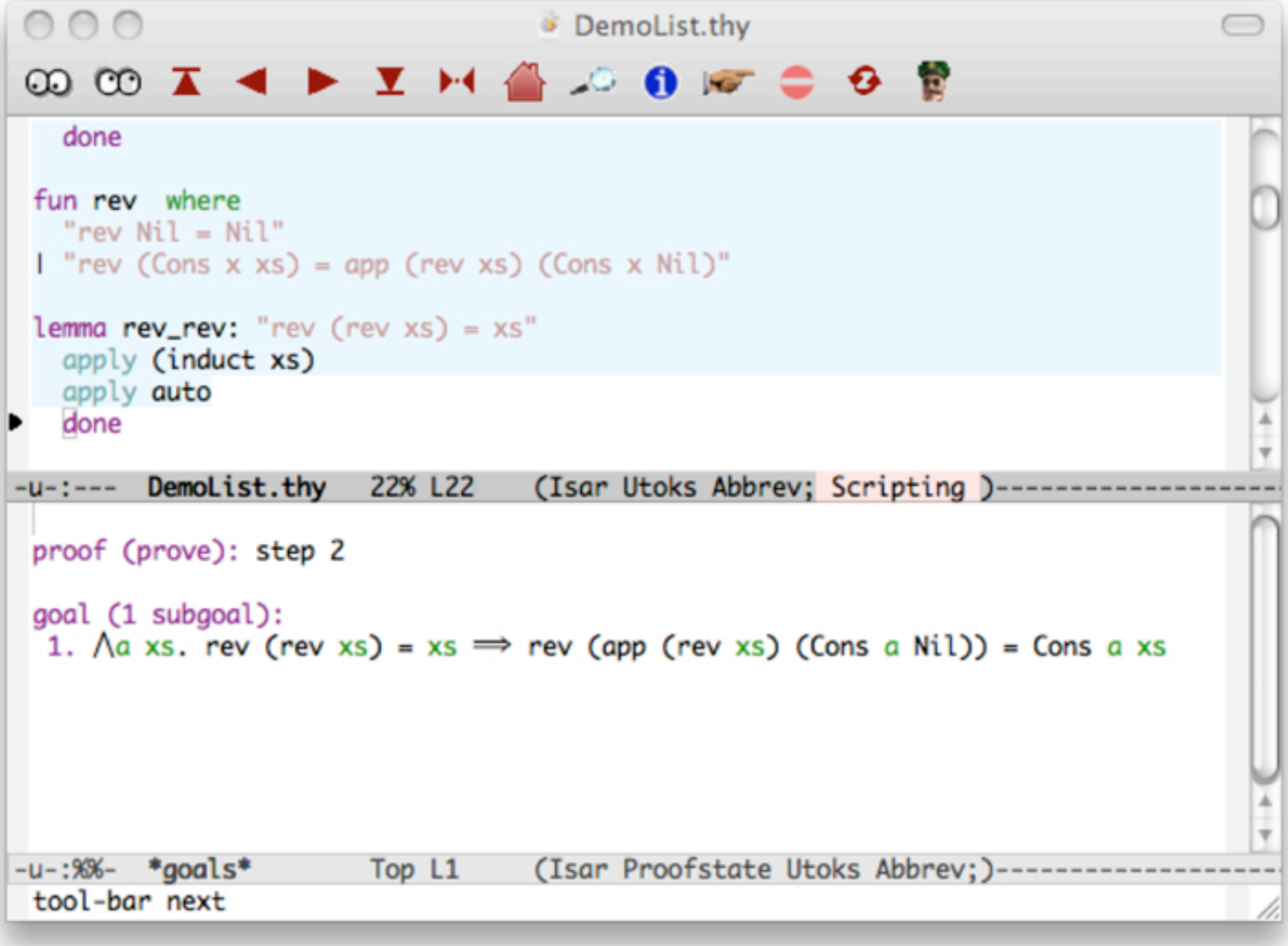
list reversal function

Can we prove both subgoals?

Stuck!



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-u-:---  DemoList.thy   22% L22    (Isar Utoks Abbrev; Scripting )------------------
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proof (prove): step 2

goal (1 subgoal):
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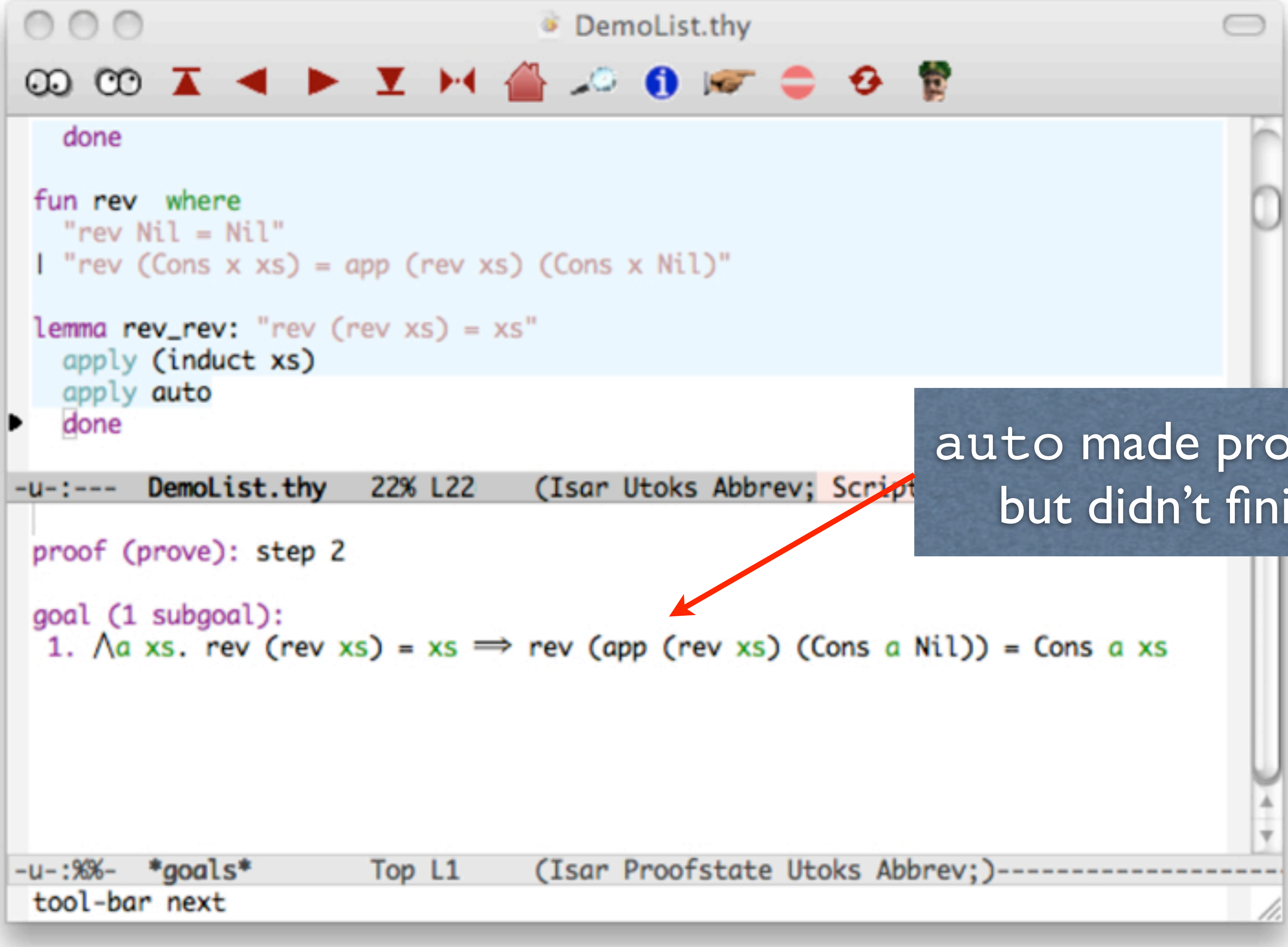
-u-:%%-  *goals*        Top L1     (Isar Proofstate Utoks Abbrev;)------------------
tool-bar next
```

Stuck!



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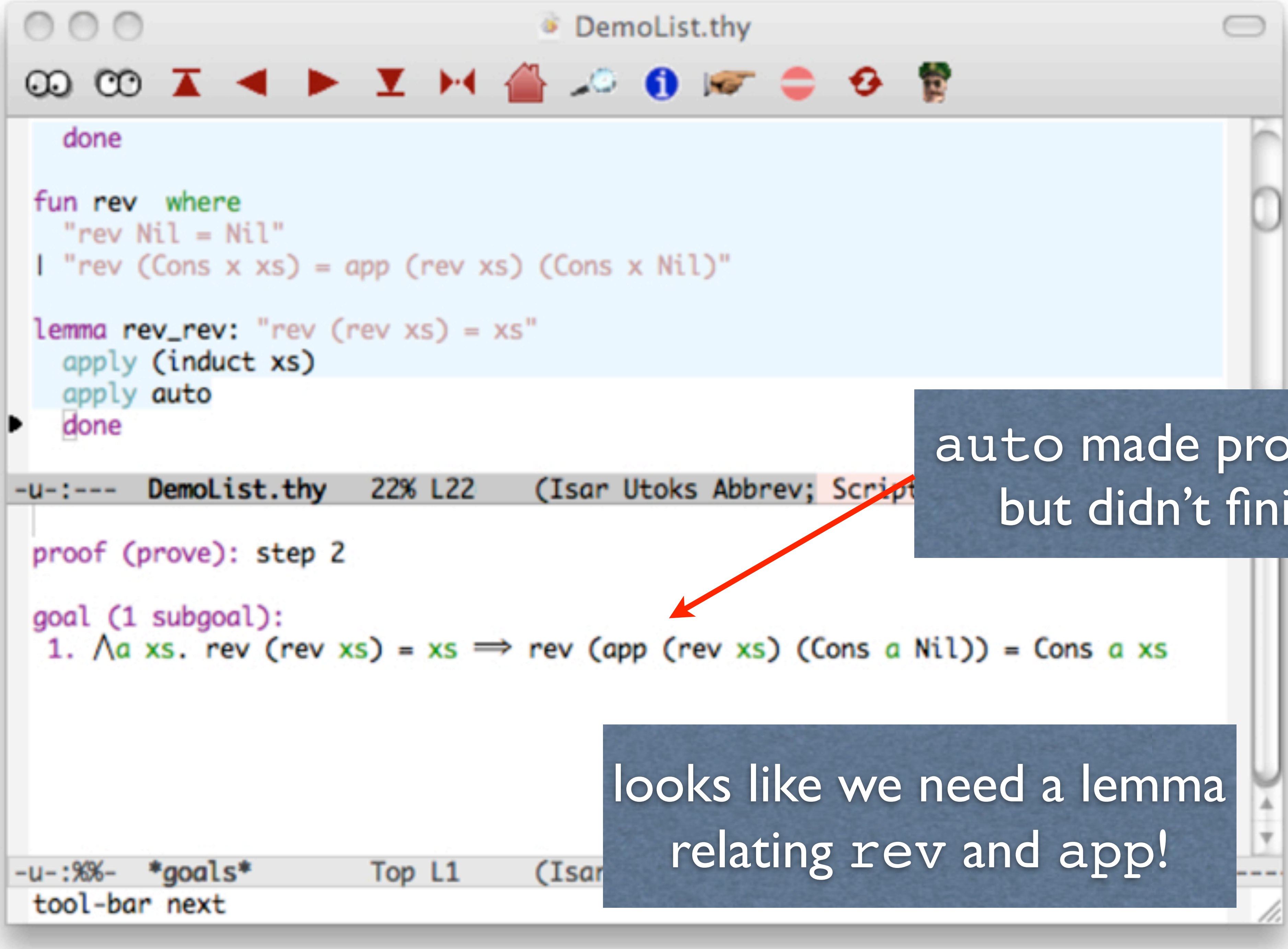
```
proof (prove): step 2

goal (1 subgoal):
 1. ⋀a xs. rev (rev xs) = xs ⟹ rev (app (rev xs) (Cons a Nil)) = Cons a xs
```

```
-u-:%%-  *goals*     Top L1    (Isar Proofstate Utoks Abbrev;)-------------------
tool-bar next
```

auto made progress
but didn't finish

Stuck!



```
  done

fun rev  where
  "rev Nil = Nil"
| "rev (Cons x xs) = app (rev xs) (Cons x Nil)"

lemma rev_rev: "rev (rev xs) = xs"
  apply (induct xs)
  apply auto
  done
```

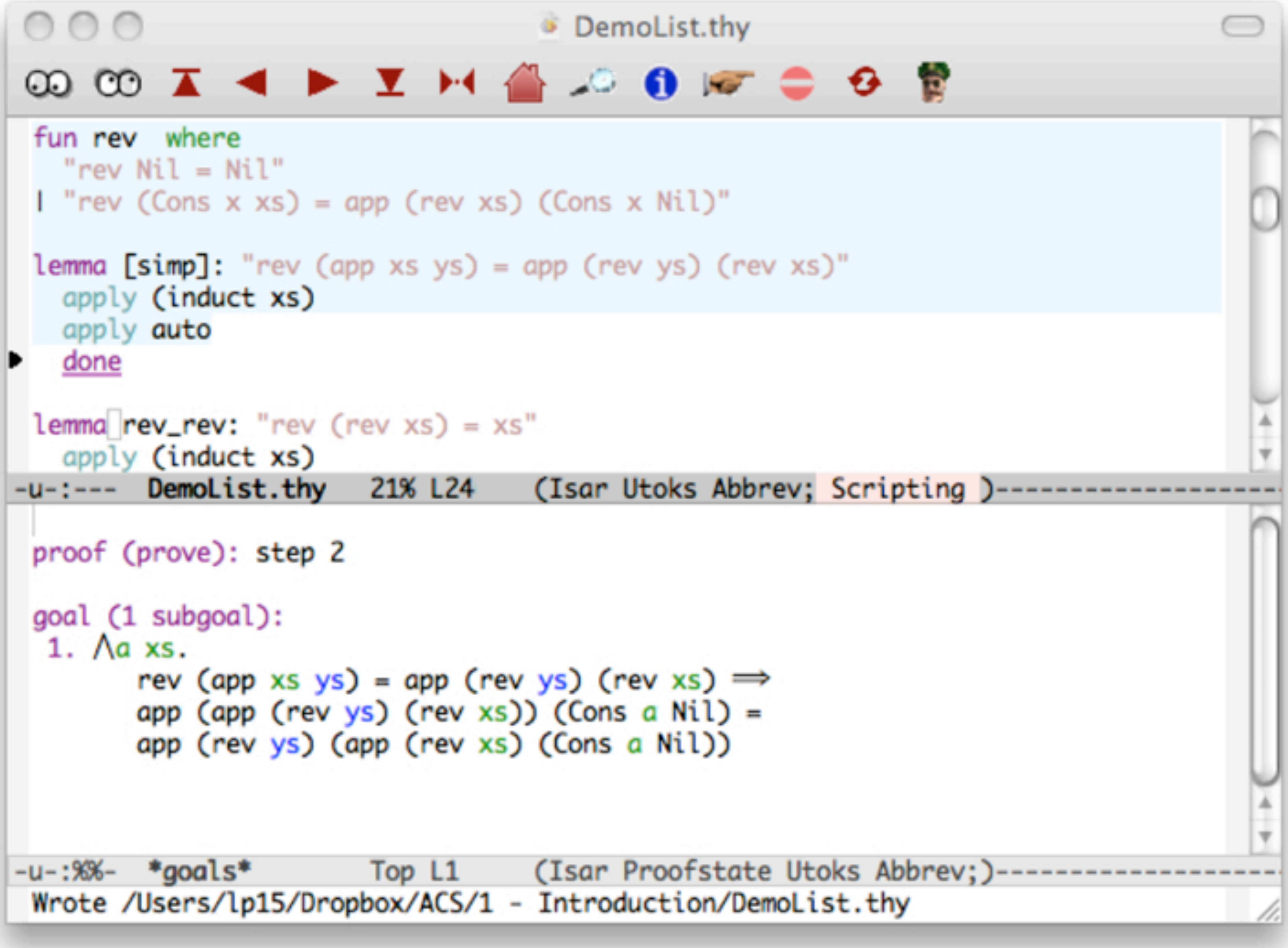
```
proof (prove): step 2

goal (1 subgoal):
 1. ⋀a xs. rev (rev xs) = xs ⟹ rev (app (rev xs) (Cons a Nil)) = Cons a xs
```

auto made progress but didn't finish

looks like we need a lemma relating `rev` and app!

Stuck Again!



```
fun rev  where
  "rev Nil = Nil"
| "rev (Cons x xs) = app (rev xs) (Cons x Nil)"

lemma [simp]: "rev (app xs ys) = app (rev ys) (rev xs)"
  apply (induct xs)
  apply auto
  done

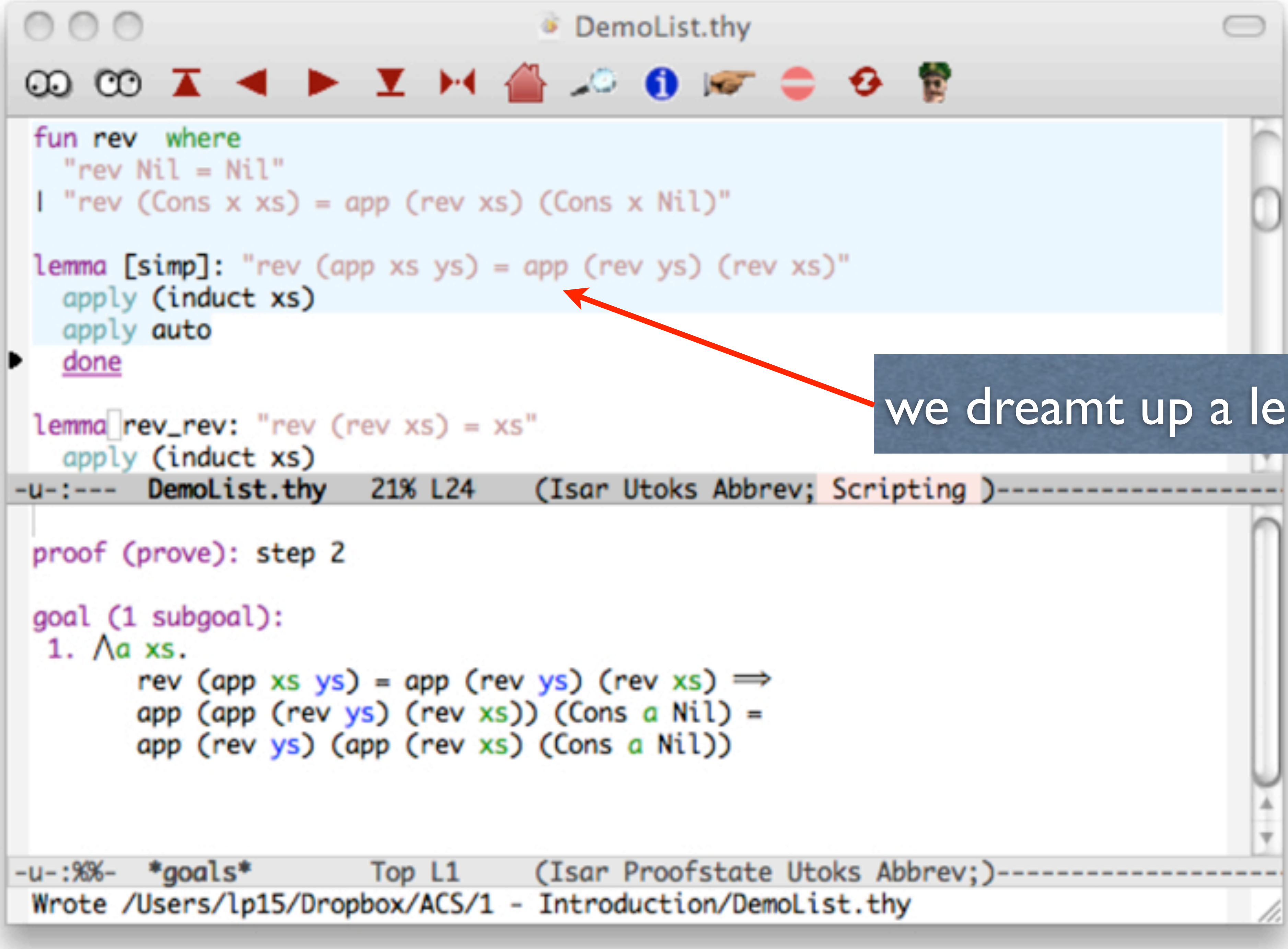
lemma rev_rev: "rev (rev xs) = xs"
  apply (induct xs)
-u-:---  DemoList.thy    21% L24     (Isar Utoks Abbrev; Scripting )------------------
```

```
proof (prove): step 2

goal (1 subgoal):
 1. ⋀a xs.
       rev (app xs ys) = app (rev ys) (rev xs) ⟹
       app (app (rev ys) (rev xs)) (Cons a Nil) =
       app (rev ys) (app (rev xs) (Cons a Nil))

-u-:%%-  *goals*        Top L1     (Isar Proofstate Utoks Abbrev;)------------------
Wrote /Users/lp15/Dropbox/ACS/1 - Introduction/DemoList.thy
```

Stuck Again!



```
fun rev  where
  "rev Nil = Nil"
| "rev (Cons x xs) = app (rev xs) (Cons x Nil)"

lemma [simp]: "rev (app xs ys) = app (rev ys) (rev xs)"
  apply (induct xs)
  apply auto
  done

lemma rev_rev: "rev (rev xs) = xs"
  apply (induct xs)
-u-:---  DemoList.thy   21% L24    (Isar Utoks Abbrev; Scripting )------------------

proof (prove): step 2

goal (1 subgoal):
 1. ⋀a xs.
       rev (app xs ys) = app (rev ys) (rev xs) ⟹
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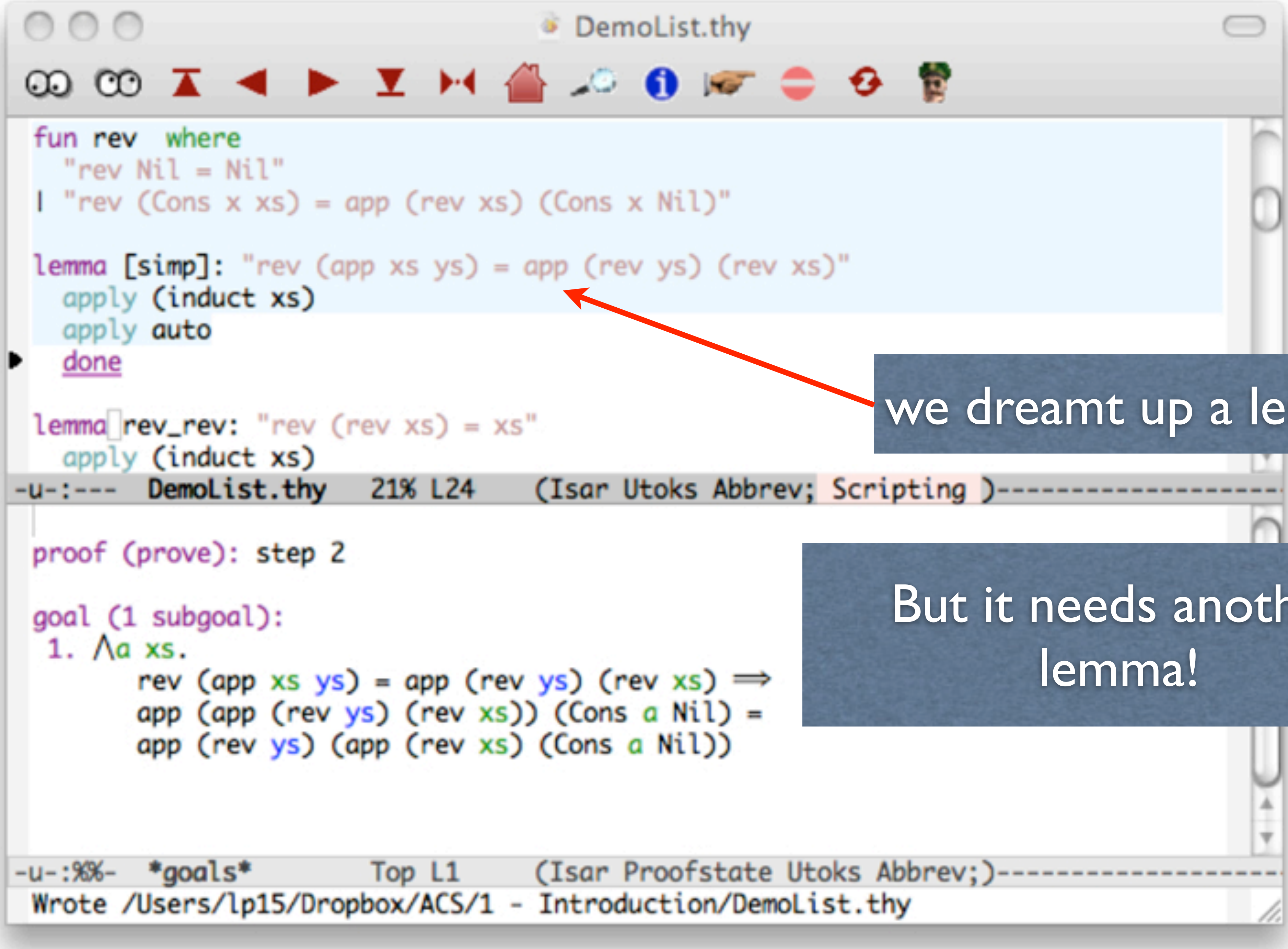
       app (rev ys) (app (rev xs) (Cons a Nil))

-u-:%%-  *goals*        Top L1     (Isar Proofstate Utoks Abbrev;)------------------
Wrote /Users/lp15/Dropbox/ACS/1 - Introduction/DemoList.thy
```

we dreamt up a lemma...

Stuck Again!



```
fun rev  where
  "rev Nil = Nil"
| "rev (Cons x xs) = app (rev xs) (Cons x Nil)"

lemma [simp]: "rev (app xs ys) = app (rev ys) (rev xs)"
  apply (induct xs)
  apply auto
  done

lemma rev_rev: "rev (rev xs) = xs"
  apply (induct xs)
-u-:---  DemoList.thy   21% L24    (Isar Utoks Abbrev; Scripting )------------------
```

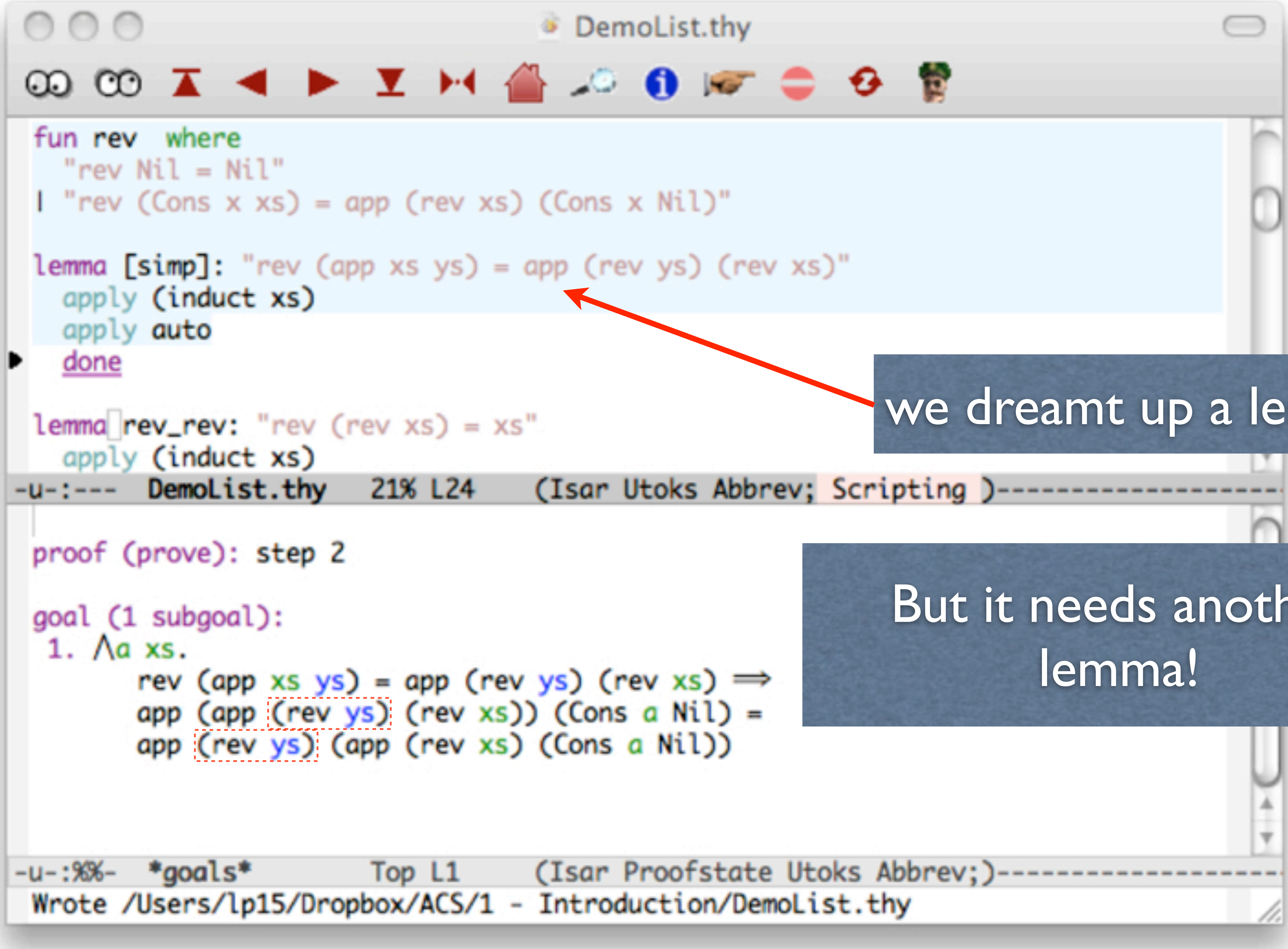
```
proof (prove): step 2

goal (1 subgoal):
 1. ⋀a xs.
       rev (app xs ys) = app (rev ys) (rev xs) ⟹
       app (app (rev ys) (rev xs)) (Cons a Nil) =
       app (rev ys) (app (rev xs) (Cons a Nil))
-u-:%%-  *goals*        Top L1     (Isar Proofstate Utoks Abbrev;)------------------
Wrote /Users/lp15/Dropbox/ACS/1 - Introduction/DemoList.thy
```

we dreamt up a lemma...

But it needs another lemma!

Stuck Again!



```
fun rev  where
  "rev Nil = Nil"
| "rev (Cons x xs) = app (rev xs) (Cons x Nil)"

lemma [simp]: "rev (app xs ys) = app (rev ys) (rev xs)"
  apply (induct xs)
  apply auto
  done

lemma rev_rev: "rev (rev xs) = xs"
  apply (induct xs)
-u-:---  DemoList.thy   21% L24    (Isar Utoks Abbrev; Scripting )------------------

proof (prove): step 2

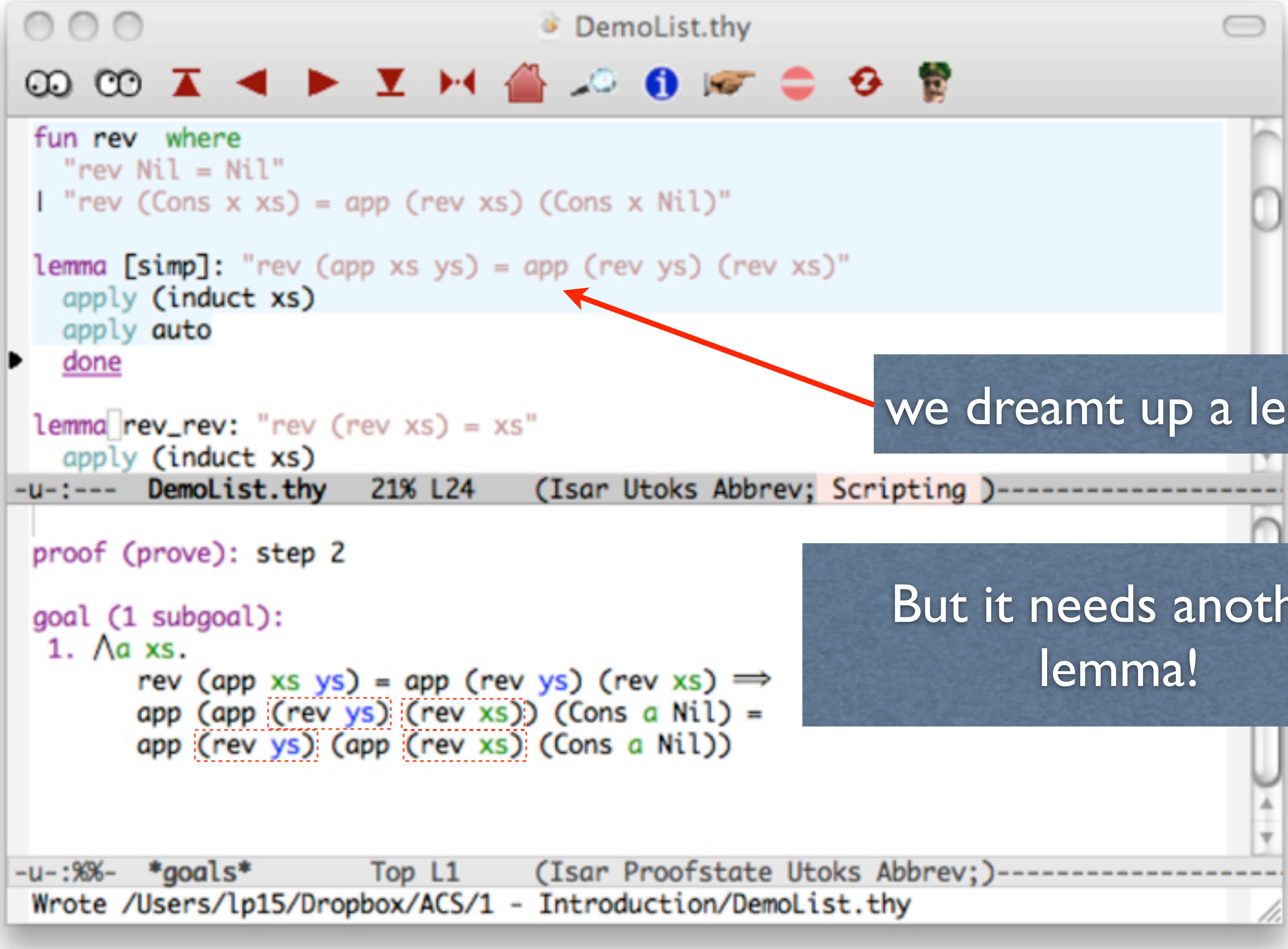
goal (1 subgoal):
 1. ⋀a xs.
       rev (app xs ys) = app (rev ys) (rev xs) ⟹
       app (app (rev ys) (rev xs)) (Cons a Nil) =
       app (rev ys) (app (rev xs) (Cons a Nil))

-u-:%%-  *goals*        Top L1     (Isar Proofstate Utoks Abbrev;)------------------
Wrote /Users/lp15/Dropbox/ACS/1 - Introduction/DemoList.thy
```

we dreamt up a lemma...

But it needs another lemma!

Stuck Again!



```
fun rev  where
  "rev Nil = Nil"
| "rev (Cons x xs) = app (rev xs) (Cons x Nil)"

lemma [simp]: "rev (app xs ys) = app (rev ys) (rev xs)"
  apply (induct xs)
  apply auto
  done

lemma rev_rev: "rev (rev xs) = xs"
  apply (induct xs)
-u-:---  DemoList.thy   21% L24    (Isar Utoks Abbrev; Scripting )-----------------

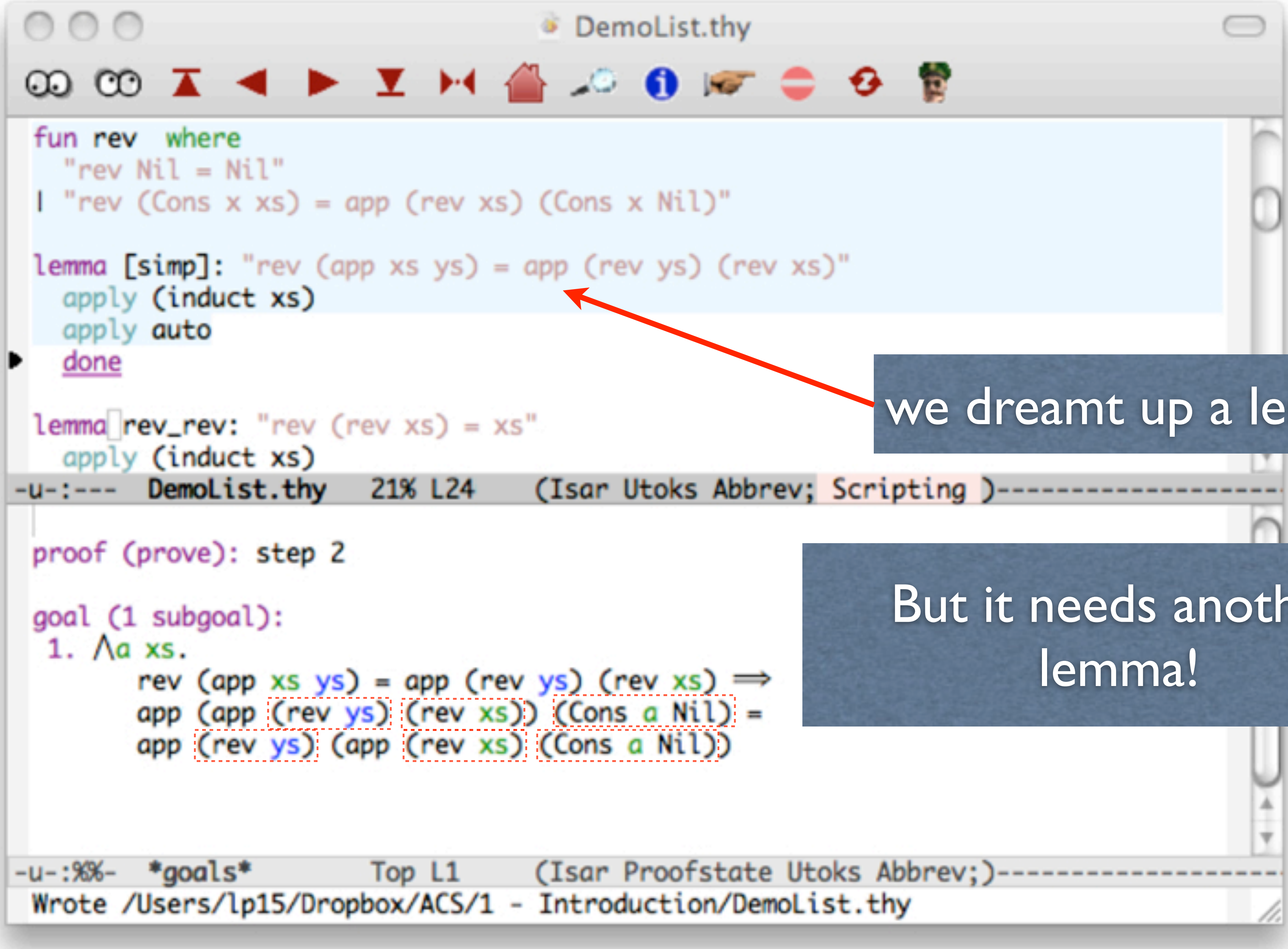
proof (prove): step 2

goal (1 subgoal):
 1. ⋀a xs.
       rev (app xs ys) = app (rev ys) (rev xs) ⟹
       app (app (rev ys) (rev xs)) (Cons a Nil) =
       app (rev ys) (app (rev xs) (Cons a Nil))
-u-:%%-  *goals*        Top L1     (Isar Proofstate Utoks Abbrev;)-----------------
Wrote /Users/lp15/Dropbox/ACS/1 - Introduction/DemoList.thy
```

we dreamt up a lemma...

But it needs another lemma!

Stuck Again!



```
fun rev  where
  "rev Nil = Nil"
| "rev (Cons x xs) = app (rev xs) (Cons x Nil)"

lemma [simp]: "rev (app xs ys) = app (rev ys) (rev xs)"
  apply (induct xs)
  apply auto
  done

lemma rev_rev: "rev (rev xs) = xs"
  apply (induct xs)
-u-:---  DemoList.thy   21% L24    (Isar Utoks Abbrev; Scripting )------------------

proof (prove): step 2

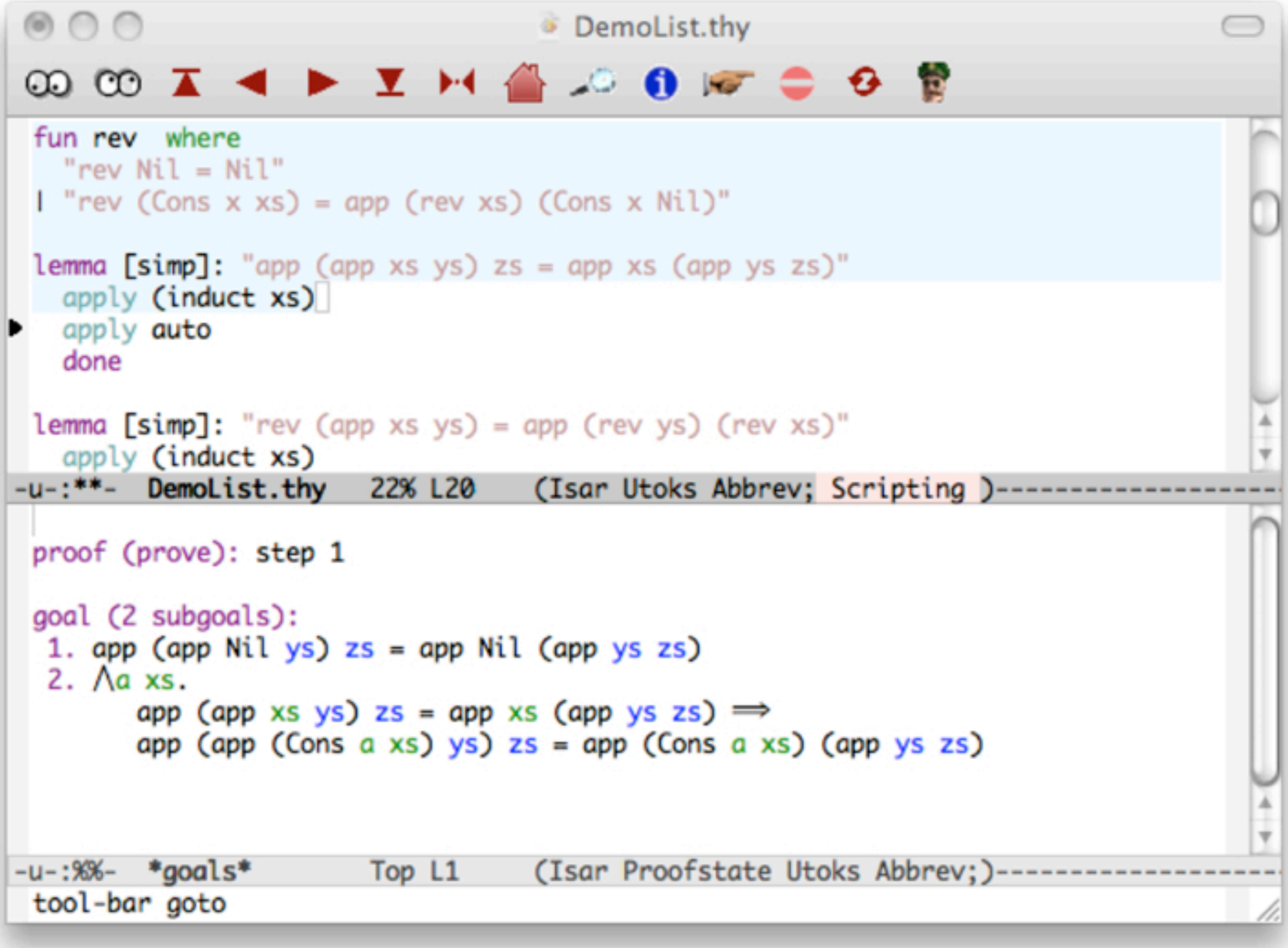
goal (1 subgoal):
 1. ⋀a xs.
       rev (app xs ys) = app (rev ys) (rev xs) ⟹
       app (app (rev ys) (rev xs)) (Cons a Nil) =
       app (rev ys) (app (rev xs) (Cons a Nil))

-u-:%%-  *goals*        Top L1     (Isar Proofstate Utoks Abbrev;)------------------
Wrote /Users/lp15/Dropbox/ACS/1 - Introduction/DemoList.thy
```

we dreamt up a lemma...

But it needs another lemma!

The Final Piece of the Jigsaw



```
fun rev  where
  "rev Nil = Nil"
| "rev (Cons x xs) = app (rev xs) (Cons x Nil)"

lemma [simp]: "app (app xs ys) zs = app xs (app ys zs)"
  apply (induct xs)
  apply auto
  done

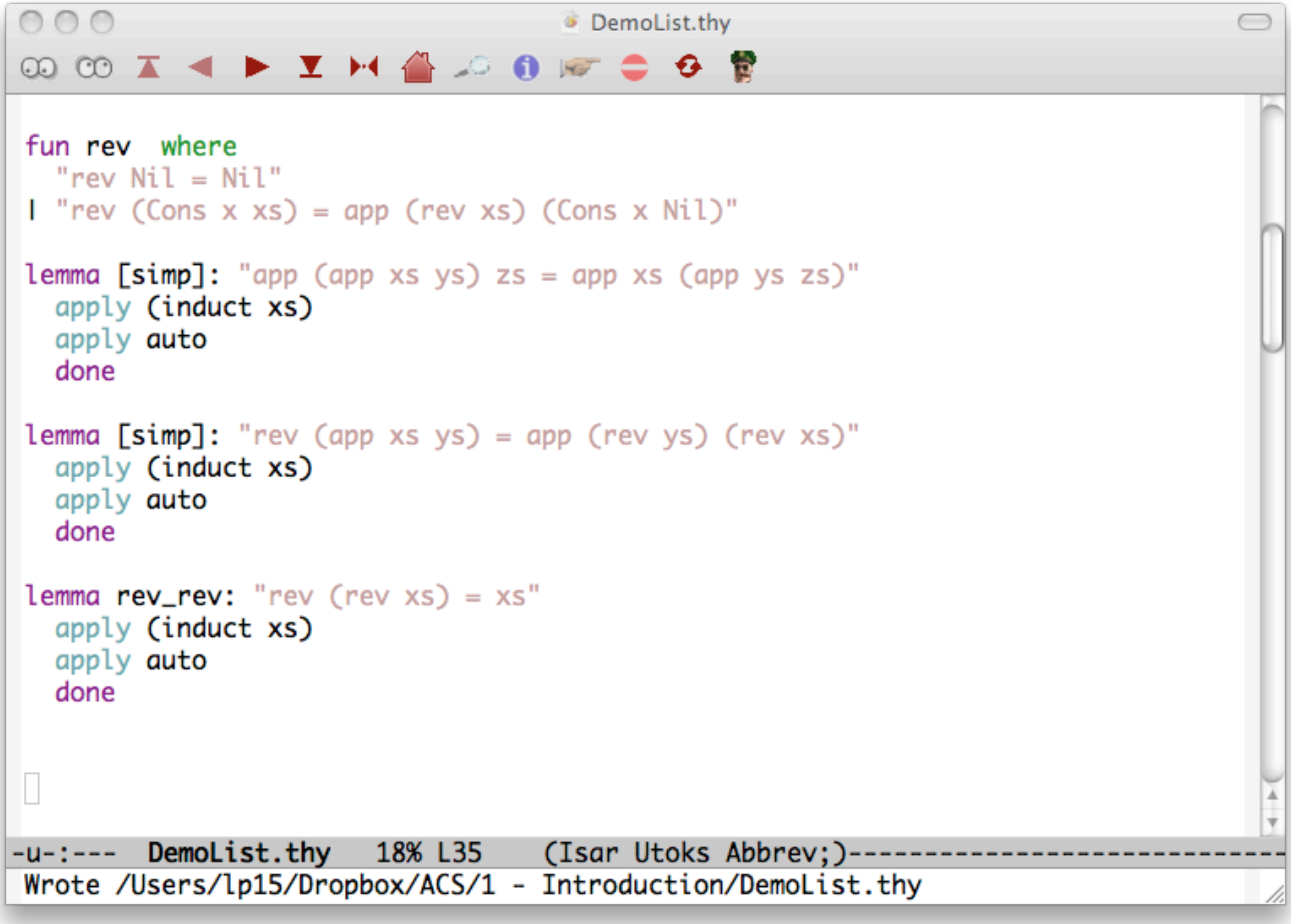
lemma [simp]: "rev (app xs ys) = app (rev ys) (rev xs)"
  apply (induct xs)
-u-:**-  DemoList.thy    22% L20     (Isar Utoks Abbrev; Scripting )------------------
```

```
proof (prove): step 1

goal (2 subgoals):
 1. app (app Nil ys) zs = app Nil (app ys zs)
 2. ⋀a xs.
       app (app xs ys) zs = app xs (app ys zs) ⟹
       app (app (Cons a xs) ys) zs = app (Cons a xs) (app ys zs)

-u-:%%-  *goals*         Top L1      (Isar Proofstate Utoks Abbrev;)------------------
tool-bar goto
```

The Finished Proof



```
fun rev  where
  "rev Nil = Nil"
| "rev (Cons x xs) = app (rev xs) (Cons x Nil)"

lemma [simp]: "app (app xs ys) zs = app xs (app ys zs)"
  apply (induct xs)
  apply auto
  done

lemma [simp]: "rev (app xs ys) = app (rev ys) (rev xs)"
  apply (induct xs)
  apply auto
  done

lemma rev_rev: "rev (rev xs) = xs"
  apply (induct xs)
  apply auto
  done
```

```
-u-:---  DemoList.thy   18% L35    (Isar Utoks Abbrev;)---------------------------
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```